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Disturbances in piezo-quartz transducers subjected to a flow of current in the semiconducting boundary layer

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Abstract : The mechanical responses in piezoelectric plate transducers (quartz) with rigid backing when the transducer is subjected to a flow of current in the semiconducting boundary layer have been investigated in the present study. The semiconducting layer is thinner than the quartz plate by at least one order of magnitude. It is found that the order of disturbance in case of a thin layer of intrinsic semiconductor InSb (at the temperature $T' = 77$ K) covering the piezo-quartz is very high compared to n-type As-doped germanium of an extrinsic semiconductor at $T' = 300$ K.

Keywords : Piezoelectricity, Electron stream, Mechanical disturbance.

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1. Introduction

The studies in the disturbances of a piezoelectric material from the stand point of mechanics of continuous media have been initiated by [1-4]. The effect of an electron stream or of a conducting layer on the electromechanical disturbances have engaged a number of researchers [5-7] by a number of decades The present study seeks to investigate the response in piezoelectric transducers (quartz) subjected to a flow of current in the semiconducting boundary layer. These types of problems are very much used in different branches of Acoustic Engineering particularly in piezoelectric amplifiers [8,9] and in different biomedical applications [10].

2. The Problem and the Fundamental Equations

Let us consider a piezoelectric transducer, piezo-quartz (X-cut) acted on by an electron stream flowing outside it. In order to avoid the difficulty of getting the very small distance between the electron stream and the piezo-quartz, we cover the surface of the latter with a semi-conducting layer in which motion of electric charges is produced by applying a voltage. The layer will play the role of an electron (or hole) guide. The choice of the semiconductor depends on the required density of carriers. We have considered the case of germanium (low concentration of carriers) of the n-type extrinsic semiconductor with arsenic at room temperature T′ = 300K and a thin layer of intrinsic semiconductor InSb (high mobility of electrons) at the temperature of liquid nitrogen T′ = 77K covering the piezo-quartz. The semi-conducting layer is thinner than the quartz plate by at least one order of magnitude. The transducer is rigidly backed at both ends [7,8]. The influence of internal and external damping is disregarded [11,12]. The influence will be greater than in the quartz itself as a result of interaction between the semi-conducting layer and the piezo-quartz. We have taken X-axis is along the normal to the surface of the transducer in the upward direction, Y-axis is along the length and Z-axis along with these two axes form a rectangular system .

We now, proceed to derive the differential equation to calculate the mechanical response with the fundamental equations.

The equations representing plane compressional wave propagation in the x-direction in a piezoelectric material [2,5] are given by

$$T = c \frac{\delta\xi}{\delta x} - hD \qquad (1)$$

$$D = E\varepsilon + h\varepsilon \frac{\delta\xi}{\delta x} \qquad (2)$$

where, c - elastic constant; ξ - the displacement in the x direction;
T - normal stress in the thickness direction; E - electric field strength; D - electric flux density in the x-direction; ε - dielectric constant and h - the piezoelectric constant of the material.

The equation of motion is

$$\rho\ \delta^2\xi/\delta t^2 = \delta T/\delta x \qquad (3)$$

where ρ is the density of the material.

The current in the semi-conducting layer is expressed as

$$J = qn_c\mu E_c + qD_n\ \delta n_c/\delta x \qquad (4)$$

where, J = electron current density, n_c total number of electrons in the conducting band, E_c = total electric field in the semiconductor, q = electron charge (absolute value), μ = mobility of electrons and D_n = Diffusion constant = $kT'\mu/q$ (5) where T′ is temperature in absolute value. The interaction constant between the disturbed electric field $\tilde{E}_s$ in the semiconductor and that $\tilde{E}$ in the transducer is defined as $\tilde{E}_s/\tilde{E} = \alpha$ (6)

Then the total field in the semiconductor is expressed by the equation $E = E_o + \Delta E$ (7) where E_o = constant outer field and ΔE = perturbed field, which is composed of the perturbed field in semiconductor and in the transducer. So that

$$\Delta E = E_s - \alpha E \tag{8}$$

Denoting now by $qn_c = \rho_{ec}$ the electric density in the semiconductor, the velocity of the electron stream can be related to the electric field by the equation $-\rho_{ec}v_c = \mu\rho_{ec}E_c$. (9)

Bearing in mind that, $\rho_{ec} = \rho_{oc} + f\rho_e$, $v_c = v_o + v$, $E_c = E_o + \Delta E$ and, $-v_o = \mu E_o$ we get

$$-v = \mu\Delta E = \mu(E_s - \alpha\tilde{E}) \tag{10}$$

where, f – a coefficient less than 1. The fraction f accounts for a division of space charge between the conduction band and bound states in the energy gap. The continuity condition of the charges in semiconductor is $\text{div } j - \delta\rho_{ec}/\delta t = 0$ (11)

which after linearization assumes the form

$$\delta\rho/\delta t + fv_o\delta\rho_e/\delta x + \rho_{oe}\delta v/\delta x - fD_n\delta^2\rho_e/\delta x^2 \tag{12}$$

The divergence condition in the transducer is

$$\mathrm{div}D = \varepsilon\delta\bar{E}/\delta x \tag{13}$$

where $\tilde{E}$ is the sourcelss field of the action on the transducer of the electron stream in the semiconductor.

Now from Eqs. (2) and (13) it follows that

$$\delta\tilde{E}/\delta x = \delta E/\delta x + h\ \delta^2\xi/\delta x^2$$

Therefore $$\tilde{E} = E + h\ \delta\xi/\delta x \tag{14}$$

Next from the continuity condition of tangential field E in the transducer and the semiconductor, we find that

$$E_s - \alpha\tilde{E} = \tilde{E} - h\ \delta\xi/\delta x$$

or, $$\tilde{E} = [E_s + h\ \delta\xi/\delta x]/1+\alpha \tag{15}$$

Again divergence condition in the semiconductor is

$$\varepsilon_s\delta E_s/\delta x = -\rho_e \tag{16}$$

Therefore $$\delta E/\delta x = -[\rho_e\eta/\varepsilon_s + h\alpha\ \delta^2\xi/\delta x^2]/1+\alpha \tag{17}$$

where η is the ratio of the cross sectional area of the semiconducting layer to that of the transducer.

Now from Eqs. (2), (3) and (17) we get

$$\delta^2\xi/\delta t^2 - a^2\delta^2\xi/\delta x^2 - h\varepsilon\rho_e\eta/\rho(1+\alpha)\varepsilon_s = 0 \tag{18}$$

where, $$a^2 = [c - h^2\varepsilon/1+\alpha]/\rho$$

From Eqs. (10), (12) and (17) we get

$$\rho_e + fv_o\delta\rho_e/\delta x + \rho_{oe}\mu\eta\rho_e/\varepsilon_s(1+\alpha)$$
$$- fD_n\delta^2\rho_e/\delta x^2 + \rho_{oe}\mu\alpha h\delta^2\xi/\delta x^2/(1+\alpha) = 0 \tag{19}$$

Now. eliminating ρ_e from Eqs. (18) and (19), we get

$$[\delta/\delta t + fv_o\delta/\delta x + \rho_{oe}\mu\eta/\varepsilon_s(1+\alpha) - fD_n\delta^2/\delta x^2]$$
$$\times [\delta^2\xi/\delta t^2 - a^2\delta^2\xi/\delta x^2]$$
$$+ \rho_{oe}(h/1+\alpha)^2\mu\alpha\eta\varepsilon\ \delta^2\xi/\delta x^2/\varepsilon_s\rho = 0 \tag{20}$$

3. Solution of the Problem

Taking Laplace transform of Eq. (20) and considering $\bar{\xi} = \exp(mx)$ be a trial solution and for small values of t it takes the form

$$fD_n m^2 - fv_o m - (p + \rho_{oe}\mu\eta/\varepsilon_s(1+\alpha)) = 0 \tag{21}$$

where p is the Laplace transform parameter.

Then the general solution is

$$\bar{\xi} = A\exp(m_1 x) + B\exp(m_2 x) \tag{22}$$

where m_1 and m_2 are the roots of the Eq. (21) and A, B are constants to be determined from boundary conditions.

For a rigidly backed transducer we assume the stresses and displacements at the ends are continuous. To obtain the solution of the problem we attach to the extremities x = 0 and x = X two mechanical systems and denote the corresponding quantities by the symbols 1 and 2. Accordingly, the boundary conditions become

$$(F_1)_0 = (F)_0, \quad (F_2)_X = (F)_X, \quad (\xi_1)_0 = (\xi)_0, \quad (\xi_2)_X = (\xi)_X$$

where F_1 and F_2 are the values of F(stress) and ξ_1 and ξ_2 are the values of ξ (displacement) at the two boundaries, i.e., at x=0 and x=X respectively.

From (1) we get $\bar{T} + h\bar{D} = c\delta\bar{\xi}/\delta x$. Applying Gauss Law at the surface of the transducer we get $\bar{D}=\bar{Q}/yz$ and $\bar{F}=\bar{T}yz$. From above two, we get

$$\bar{F}+h\bar{Q} = yzc\delta\bar{\xi}/\delta x$$

y and z are the dimension of the piezoquartz bar. Voltage across the transducer is given by

$$\int_0^X \bar{E}dx = -\{(\bar{V})_X-(\bar{V})_0\} = \bar{V}(\text{say}) = -h\{(\bar{\xi})_X-(\bar{\xi})_0\}+\bar{Q}X/\varepsilon yz$$

or $$\bar{V} = -h\{(\bar{\xi})_X - (\bar{\xi})_o\} + \bar{Q}/C_o$$

where $C_o = \varepsilon yz/X$ is called the static capacitance of the transducer.

Taking into consideration impulsive voltage function $V = V_o\delta(t)$ where $\delta(t)$ is the Dirac delta function, we get $\bar{V} = V_o/p$ and by using conditions of continuity of forces and displacements as stated above, we get

$$\bar{\xi}_o = V_o\sqrt{fD_n}\,/2h\,p\sqrt{p+a'}$$
$$+\ Q\sqrt{fD_n}\,\{h/yzc - 1/hC_o\}/2\sqrt{p+a'}$$

where, $$a' = fV_o^2/4D_n + \rho_{oe}\mu\eta/\varepsilon_s(1+\alpha)$$

To get its inverse transform, the mechanical disturbance in the transducer, we proceed [12,13]

$$\xi_o = V_o\sqrt{fD_n\,\pi}\ .\ \mathrm{erf}(\sqrt{a't})/2h\sqrt{a'}$$
$$+\ Q\sqrt{fD_n}\,\{h/yzc - 1/hC_o\}\,\exp(-a't)t^{-1/2}/2\sqrt{\pi} \qquad (23)$$

[erf – error function]

4. Discussion

Here we have considered two examples of semiconductor covering the piezo-quartz. Let us consider i) a thin layer of the intrinsic semi-conductor InSb (at the temperature of liquid nitrogen, 77K) covering the piezo-quartz and ii) the case of n-type As-doped germanium covering the piezo-quartz at 300K. The standard values of the material constants are taken from [2,5,8]. The variation of the mechanical disturbance produced in the piezo-quartz transducer subjected to a flow of current in the semiconducting boundary layer with time is found to be parabolic in nature (Fig.1) and is the order of 10^{-8} m in case of (i) and in the case (ii), the variation of the disturbance is found to be of the order of 10^{-12}m.

The disturbances consist of two parts - an exponential function and an error function of time. Here we have considered two examples of semiconductor covering the piezo-quartz. It is interesting to note that when a piezo-quartz is covered by a thin layer of intrinsic semiconductor

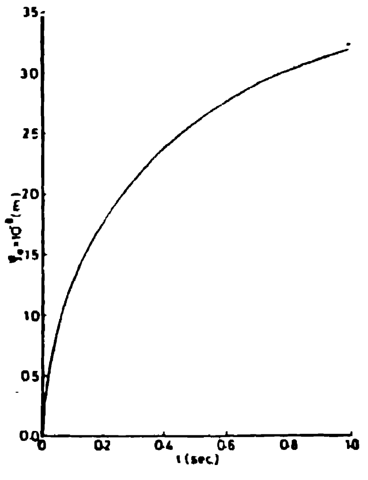

Figure 1. The variation of disturbance when covered by a thin layer of intrinsic semiconductor InSb at 77K.

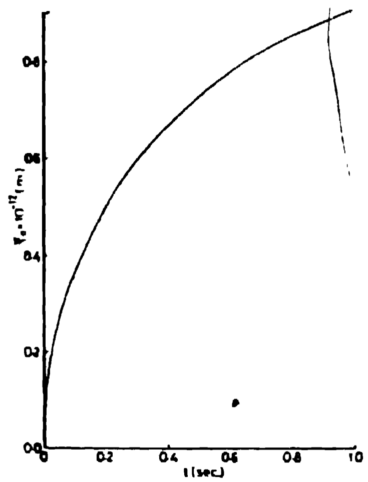

Figure 2. The variation of disturbance when covered by a thin layer of germanium with arsenic at 300K.

InSb at the temperature of liquid nitrogen the values ofthe mechanical disturbance increases almost 10^4 times than when a thin layer of germanium of the n-type extrinsic with asrsenic at room temperature covering the piezo-quartz.

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